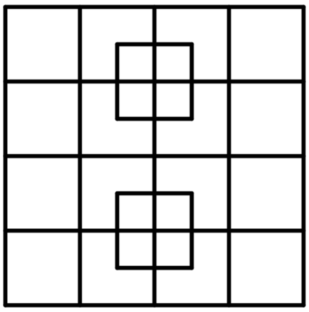
MatX 2019 Solutions

matx.p-mat.sk

28 March 2019

Problem 1
How many squares are there in the picture?



Solution 1
To eliminate counting any square twice, let's have a system where we count the squares of the same size always together. We then sum these counts and get the final answer.
1×1: 16 + 2 = 18
2×2: 9
3×3: 4
4×4: 1
Small 1×1 in the middle: 8
All together 18 + 9 + 4 + 1 + 8 = 40 squares.

Problem 2
There are two trees in a garden – a cherry tree and a pear tree. There are birds sitting on both of the trees. If one of the birds from the cherry tree flew to the pear tree, there would be twice as many birds on the pear tree as there are on the cherry tree. If one of the birds from the pear tree flew to the cherry tree, there would be the same amount of birds on both trees. How many birds are sitting on the pear tree?
Solution 2
Let's label the number of birds on each tree C and P. The two given conditions can be then written using equations as:

$$P + 1 = 2 \times (C - 1) \tag{1}$$
$$P - 1 = C + 1 \tag{2}$$

We use the second equation to get $P = C + 2$. By plugging this result into the first equation we get an equation $C + 2 + 1 = 2 \times (C - 1)$, which gives us that $C = 5$, therefore $P = 7$. There are 5 birds on the cherry tree and 7 birds on the pear tree.

Problem 3

Two classmates play a card game. Each time one of them wins, the one who lost has to give the other an apple. It is not possible to draw in the game they play. At the end of a school day, one of them has already won three games while the other has gained three apples that day. How many games did they play?

Solution 3

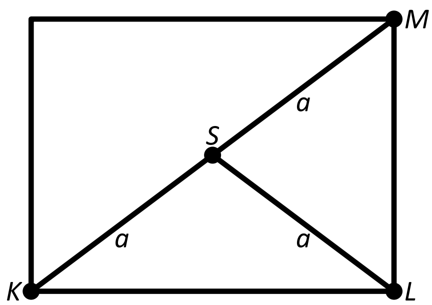
The first classmate won three games, so he had to get three apples from the other classmate. However, the other classmate gained three apples that day in total, he had to therefore win six games – three to get the three apples back and three to gain the three apples. Together they played nine games, three were won by the first classmate, six by the other.

Problem 4

Girls are playing in a rectangular garden. Karen, Lena and Monica went to its three corners while Sarah went to the centre of the garden. Karen and Lena stretched an 8 meter long blue rope between each other. Then Sarah joined them, so they stretched a 18 meter long red rope between the three of them so that it formed a triangle. Then Monica wanted to play along with them, so Lena, Monica and Sarah stretched a 16 meter long green rope between the three of them so that it formed a triangle. How long rope would they need if Sarah went to the fourth corner and they were to stretch a rope between all four of them on the circumference of the garden?

Solution 4

The picture shows how the girls stood in the garden. We know that $|KS| = |LS| = |MS|$, because all of them are half of the diagonal long. Let us call that length a. We know that $|KL| = 8$ m, $a+a+8$ m $= 18$ m, $a + a + |ML| = 16$ m. We therefore immediately conclude that $a = 5$ m. That implies $|ML| = 6$ m. The original question can be rephrased as a question on what is the circumference of the garden. Since we now know the lengths of both sides of the rectangle, we are able to calculate this: $(8+8+6+6)$ m $= 28$ m.



Problem 5

After completing a construction project, the supervisor asked the worker to cut the leftover wooden beams into smaller pieces that could be used for heating. The worker asked how much is he going to get paid for that job. So the supervisor asked him to cut a beam into three pieces and measured how long it took him. Since the supervisor knew what is the worker's hourly salary, he told him he would get 80 cents for that job. How many Euro did the worker get if he cut 5 beams into 3 pieces, 17 beams into 4 pieces, 8 beams into 5 pieces and 2 long beams into 7 pieces?

Solution 5

The tricky bit in this exercise is that we have to count the number of cuts, not the number of pieces. When the worker cut the first beam to give it a try, he cut in in three pieces but using only two cuts. Therefore the price per cut is 40 cents. The only thing left is count the total number of cuts the worker did. We can count that as (number of beams) times (number of pieces minus 1) which is altogether $5 \times 2 + 17 \times 3 + 8 \times 4 + 2 \times 6 = 105$ cuts. He got 40 cents per cut, which is 0.4 Euro, hence he got in total $105 \times 0.4 = 42$ Euro.

Problem 6

In the following exercise replace each letter by a digit, while the same letters should be replaced by the same digit and different letters should be replaced by different digits.

```
A A B B A
- C C B B
---------
A B C B A
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What is the value of the 5-digit number ABCBA?

Solution 6

We can transform the subtraction problem into an addition problem:

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A B C B A
+ C C B B
---------
A A B B A
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We see that B has to be 0 since that is the only way we can satisfy the leftmost column where A + B = A. Once we know this we see that C + C = 10, so C = 5. From this it is easy to conclude that A = 6. ABCBA is therefore 60506.

Problem 7

We call a number happy, if it is divisible by 17. We call a number good, if it is divisible by 13. How many natural numbers greater than zero and smaller than 20000 are happy and good at the same time?

Solution 7

We see that both numbers (13 and 17) are prime numbers. Therefore numbers that are both happy and good at the same time are only multiples of the product of these two numbers. The product of 13 and 17 is 221. So we just need to count how many multiples of 221 smaller than 20000 are there, i.e. how many are there between 1 and 19999. There is 19999 / 221 rounded down such numbers, which is 90.

Problem 8

We perform a perfect shuffle of 10 cards like this:

1. We place the cards in a single deck and then split it into two decks – one with the upper 5 cards and one with the lower 5 cards.
2. We merge those two decks back into a single deck by interleaving the cards from the two decks 1, 2, 1, 2, ...

(For instance 6 cards numbered 1, 2, 3, 4, 5, 6 would be ordered 1, 4, 2, 5, 3, 6 after a perfect shuffle.) We have an ordered deck of 10 cards. How many times do we have to perfectly shuffle it to get the cards back in the original order?

Solution 8

There is a general formula to count the number of perfect shuffles needed. But in this case it was easier

to write down a few orderings and notice, that we get back to the original order after six shuffles:
Original: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
First: 1, 6, 2, 7, 3, 8, 4, 9, 5, 10
Second: 1, 8, 6, 4, 2, 9, 7, 5, 3, 10
Third: 1, 9, 8, 7, 6, 5, 4, 3, 2, 10
Fourth: 1, 5, 9, 4, 8, 3, 7, 2, 6, 10
Fifth: 1, 3, 5, 7, 9, 2, 4, 6, 8, 10
Sixth: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Problem 9

John would like to measure 6 minutes using hourglasses. However, he only has one that measures 5 minutes and one that measures 4 minutes. What is the smallest number of inversions that John has to do in order to do the measurement (including the initial inversions of the hourglasses)?

Solution 9

We can't measure six minutes using only a single inversion, because that way we can measure either 4 or 5 minutes. We can measure 4, 5, 9 (we start the second one after the first one) or 1 minute (we start both at once a measure the time starting when the 4-minute one finishes). However, we still can't measure six minutes. Let's try three inversions then. This is indeed possible:

- Time 0: We invert both clocks. (2 inversions)
- Time 4: 4-minute clock finished. We start measuring the time from now, but we don't invert the 4-minute clock any more. (0 inversions)
- Time 5: 5-minute clock finished, we invert it. (1 inversion)
- Time 10: 5-minute clock finished. We measured time from the 4th to the 10th minute which is 6 minutes. (0 inversion)

Therefore we need at least three inversions.

Problem 10

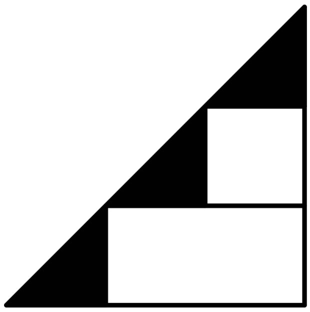
In the following equation replace each letter by a digit, while the same letters should be replaced by the same digit and different letters should be replaced by different digits.
AB – BA = 54
How many different replacements of the letters by digits are there?

Solution 10

Since the result has to be a two-digit number and AB and BA have to be both two-digit numbers (therefore at least 10), A has to be definitely equal to at least 6. We therefore need to try 4 possible values for A = 6, A = 7, A = 8 and A = 9.
A = 6: In this case B would have to be equal to 0 to satisfy the given equation. This contradicts the assignment though.
A = 7: In this case B = 1 since 71 — 17 = 54.
A = 8: In this case B = 2 since 82 — 28 = 54.
A = 9: In this case B = 3 since 93 — 39 = 54.
Therefore, the equation has three different ways how to replace the letters by digits.

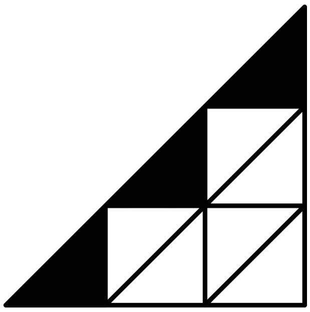
Problem 11

There is an isosceles triangle in the picture which has two rectangles inscribed in it so that the hypotenuse of the triangle is divided in equal thirds by the vertices of the rectangles. What is the ratio between the area coloured black and the area of the entire triangle?



Solution 11

If we add four extra segments into the original picture as we did in the picture below, the triangle will be divided into 9 smaller triangles of equal size. These are all of equal size since all of them have right angle and the sides that form it are all of equal lengths. Since the area coloured black covers three small triangles and the whole area of the triangle is covered by nine small triangles, the ratio of the area coloured black to the area of the entire triangle is 3 / 9 = 1 / 3.

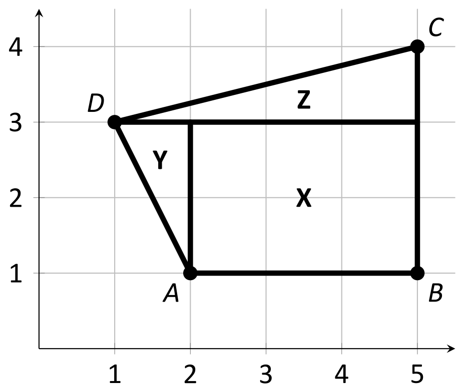


Problem 12

Luke drew points A at (2; 1), B at (5; 1), C at (5; 4) and D at (1; 3) in a coordinate grid. What is the area in cm^2 of the quadrilateral ABCD if AB is 3 cm long?

Solution 12

Let's draw the given polygon and divide it into three parts as shown in the picture below. The part X has area 3 × 2 = 6 cm^2, the part Y has area 1 × 2 / 2 = 1 cm^2, the part Z has area 4 × 1 / 2 = 2 cm^2. The total area is therefore 6 + 1 + 2 = 9 cm^2.



Problem 13

How many pairs of parallel edges does a cube have?

Solution 13

A cube has 12 edges in total with three groups of four edges that are all parallel. If we have four edges, there are $4 \times 3 / 2 = 6$ pairs of edges that are parallel with each other. A cube has therefore $6 + 6 + 6 = 18$ pairs of edges that are parallel with each other.

Problem 14

How many 6-digit palindromes are there such that the first digit is equal to the second digit plus three and the second digit is equal to the third digit plus three? Note: Palindrome is a number that reads the same backwards as forwards, for example 1234321 is a palindrome.

Solution 14

Due to the requirements on the first three digits there are only four options what numbers these three digits could be: 630, 741, 852 or 963. We therefore get four possible 6-digit palindromes: 630036, 741147, 852258 or 963369. There is one more condition we need to satisfy though, which is that the palindromes have to be divisible by six. Only the palindromes 630036 and 852258 satisfy this, there are therefore only two such numbers.

Problem 15

The year 234 was magical according to a legend. By the same legend, whenever a year is magical, also the year in that year's digit sum years will be magical (so the year $234 + (2 + 3 + 4) = 243$ is also magical). Is the year 2019 magical?

Solution 15

We could either solve this problem by writing down numbers up to 2019, or significantly faster if we are smart about it. So let's be smart about it. The rule of divisibility by nine tells us that a number is divisible by nine if and only if the sum of its digits is divisible by nine. 234 is divisible by nine, because $2 + 3 + 4 = 9$. We get the next magical year by adding 234 and the sum of its digits. Since this sum of digits is divisible by nine, the next magical year will be also divisible by nine. This will be true forever, because we will always end up with a magical year that is divisible by nine and then we will add its sum of digits to it which is also divisible by nine. Since 2019 is not divisible by nine, it can't be a magical year.

Problem 16

The points at (0; 0) and at (36; 84) in a coordinate grid are connected by a segment. How many grid points, excluding A and B, does the segment AB pass through?

Solution 16

If we reduce the fraction $36 / 84$ into its simplest form, we get $36 / 84 = 3 / 7$. Now let's draw a segment between the points (0; 0) and (3; 7) – we see it goes only through the grid points (0; 0) and (3; 7) because the greatest common divisor of 3 and 7 is only 1. However, if we stack this segment 12 times on a straight line without any gaps, we get the segment AB. This is because 12 times shifting by (3; 7) gets us to the point $(12 \times 3; 12 \times 7) = (36; 84)$. Therefore it is clear that the segment AB will go through 13 grid points (including A and B) since each segment contains one grid point + there is one extra at the end. After subtracting the two points A and B which are at the end, we get the result 11 points.

Problem 17

Two classes, 8. A and 8. B, are going for a school trip to the Peak District and their teachers are buying train tickets from London to Sheffield. Each class is accompanied by their teacher. The ticket for adults is more expensive than the ticket for children but not more than twice as much. The teacher of 8. A is going to pay 99 Pounds for his ticket and the children's tickets. The teacher of 8. B, which has four more students than 8. A, is going to pay 115 Pounds for his ticket and the children's tickets. How much does a ticket for one of the teachers cost in Pounds?

Solution 17

Let's denote c the price of the children's ticket, t the price of the teacher's ticket and s the number of students. We know that:

$$t + sc = 99 \tag{1}$$
$$t + (s + 4)c = 115 \tag{2}$$

Let's get rid of c and t by subtracting the first equation from the second one:

$$t + (s + 4)c - (t + sc) = 115 - 99$$
$$4c = 16$$

Now we know that $c = 3$, so the price of the children's ticket is 4 Euro. But we still need to find out the price of the teacher's ticket. We will use the fact that $t > c$ and at the same time t is not greater than $2c$, so the price of the teacher's ticket is somewhere between 4 and 8 Euro. That means that in the first equation the teacher had to pay for the children somewhere between 91 an 95 Euro. The price of the children's ticket is 4 Euro, so the total price has to be divisible by four. The only number between 91 and 95 that is divisible by four is 92. That implies that the price of the teacher's ticket is $99 - 92 = 7$ Euro.

Problem 18

Sisters Jane, Elisabeth, Kitty and Lydia embroider during their free time. At the beginning they had 200 pearls. During the embroidering session Jane gave Elisabeth 26 of her pearls, Elisabeth gave 36 of her pearls to Kitty, Kitty gave Lydia 32 of her pearls and Lydia gave Jane 4 of her pearls. All sisters used all pearls they had. Once they were finished, they were surprised to find out that all of them used exactly the same amount of pearls for their embroideries. How many pearls did Jane have before they started embroidering?

Solution 18

Let's write down what we know in a more concise format:

$J + E + K + L = 200$

$J \to E: 26, E \to K: 36, K \to L: 32, L \to J: 4.$

If we subtract the number of beads each sister gave from the number of beads each of them had at the beginning, we have to get the same number. It is therefore the case that:

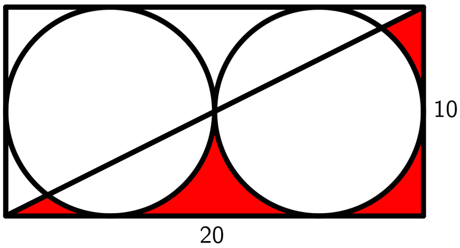
$J - 26 + 4 = E - 36 + 26 = K - 32 + 36 = L - 4 + 32$

$J - 22 = E - 10 = K + 4 = L + 28$

Since we know that all sisters used all the beads they had, it has to still be the case that the sum of all the beads at the end is 200. It must be therefore true that $4 \times (J - 22) = 200$ from which we get that $J = 72$, which means that Jane had 72 beads when she started embroidering.

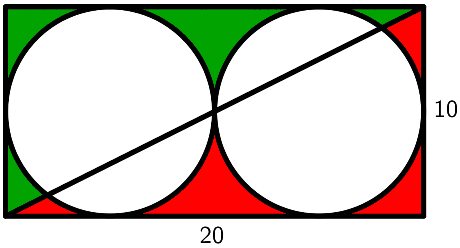
Problem 19

What is the area of the red parts of the picture in cm^2?



Solution 19

Let's colour green the parts that are neither red, neither inside the circles. Thanks to symmetry by the centre of the rectangle, the green parts are the same as the red parts and therefore their areas are the same. The area of the red parts is therefore one half of (area of the rectangle minus the area of both of the circles) = $(10 \times 20 - \pi \times 5^2 \times 2)/2 = 100 - 25\pi$, which is approximately equal to 21.46 cm^2.



Problem 20

A flea is sitting on a stretched rope and it would like to jump 10 times to get 60 cm to the right. The flea can do only 10 cm long jumps, but it can jump to the left or to the right. By how many different sequences of jumps can get the flea 60 cm to the right?

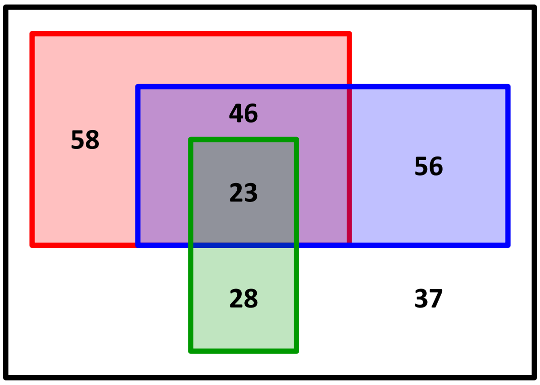
Solution 20

Since the flea needs to get 60 cm to the right using 10 jumps, it needs to do two jumps to the left (20 cm) and eight jumps to the right (80 cm). The flea can do those in different order, e.g. LLRRRRRRRR or LRRRRRRRRL (we denote L = left, R = right). We want to count how many different sequences like this are there. Since all the jumps to the left are all indistinguishable and all the jumps to the right are indistinguishable, we only need to count how many ways there are to choose two jumps to the left out of ten jumps. We have ten options to choose the first jump from and nine options to choose the second jump from. That gives us $10 \times 9 = 90$ options. However, we need to be careful since we counted each option twice, because we distinguished between the first and the second jump. To get the right result we need to divide by two, so altogether there are 45 different ways.

Problem 21

Carl was in his school's cafeteria and he asked his classmates which of the three meals on the menu they like. He wrote down the results in the diagram below. Decide which of the following statements are false based on the diagram. Your answer should be the sum of the numbers the false statements are numbered with.

1) If somebody likes goulash and halušky, they also like fried cheese.
2) More than a half of people like halušky.
4) 37 people don't have exactly one meal they like.
8) More than 84 % of people like at least one meal on the menu.
16) Twice as many people like fried cheese as goulash.
32) If the cafeteria stopped offering halušky, more than a half of people would not have a meal they like to choose.
64) If somebody likes halušky and fried cheese, they also like goulash.
128) Less than a half of people have exactly one meal they like.



Legend:
Black: Everyone
Red: Like halušky
Blue: Like fried cheese
Green: Like goulash

Solution 21

Let's go over the statements one by one and determine which are true and which are not. Then we just need to sum the numbers of the false ones.

1) True. There are no people who like only goulash and halušky but not fried cheese.
2) True. To be precise, (58 + 46 + 23) / (58 + 46 + 23 + 56 + 28 + 37) = 51 % of people like halušky.
4) False. Everyone who likes at least two meals falls into this category, therefore the number of such people would be 37 + 23 + 46.
8) True. To be precise, (58 + 46 + 23 + 56 + 28) / (58 + 46 + 23 + 56 + 28 + 37) = 85 % of people like at least one of the meals on offer.
16) False. 46 + 23 + 56 = 125 people like fried cheese, 23 + 28 = 51 like goulash. 51 × 2 = 102, which is not equal to 125.
32) False. If they stopped offering halušky, only those, who like only halušky would have nothing to choose. There is only 58 such people, which is only 23 % of people.
64) False. There are 46 people who like both but not goulash.
128) False. (58 + 56 + 28) / (58 + 46 + 23 + 56 + 28 + 37) = 57 %.

The sum of all the false statements is 4 + 16 + 32 + 64 + 128 = 244.

Problem 22

Find the largest number such that all of its pairs of consecutive digits are two-digit squared integers. Note: A squared number is a number that can be written as a product of two equal integers.

Solution 22

Here are all squares of natural numbers that have exactly two digits: 16, 25, 36, 49, 64, 81. We need to form the biggest number we are searching for using these numbers. Let's try how big would the number be if it began with all the possible squares listed above: 1649, 25, 3649, 49, 649, 81649. All numbers end with a five or a nine since there is no square with two digits that begins with a five or a nine. The largest of these numbers is 81649.

Problem 23

An old man died and left 10 000 Pounds to be divided among his heirs – his three sons and their wives. He split the heritage by each heirs' merits in the following way:

The wives got together 3 960 Pounds, while Ivana got 100 Pounds more than Annette and Danielle got 100 Pounds more than Ivana. Peter got twice as much as his wife, Eric got exactly the same amount as his wife and George got 1.5 times as much as his wife.

Who is Annette's husband?

Solution 23

Let's label the amount of money each person got by their initial. We know that A + I + D = 3960 and also I = A + 100, D = I + 100. We can plug the second equation into the third equation and we get that D = A + 200. If we plug this and the second equation into the first equation, we get that A + A + 100 + A + 200 = 3A + 300 = 3960. That implies A = 1220, I = 1320 and D = 1420.

There are six possibilities of who is married to whom:

1. A + P, I + E, D + M
2. A + P, I + M, D + E
3. A + E, I + M, D + P
4. A + E, I + P, D + M
5. A + M, I + P, D + E
6. A + M, I + E, D + P

We calculate how much would every couple inherit. Only in the third case: A + E = 2440, I + M = 3300, D + P = 4200 is the sum of all the inheritances equal to 10000. Annette is therefore married to Eric.

Problem 24

There are two equal mugs, the one on the left is filled with milk and the one on the right is filled with tea. We pour a spoon of tea from the mug on the right over to the mug on the left and stir it. Now the mug on the left has a mixture with a lot of milk and little tea. We now pour a spoon of that liquid to the mug on the right. Both mugs now have the same amount of liquid. Is there more tea in milk in the mug on the left or is there more milk in tea in the mug on the right?

A) Left

B) Right

C) Both are the same

D) This cannot be determined

Solution 24

At the beginning we have x litres of milk in the mug on the left, x litres of tea in the mug on the right and a spoon of size s litres. Let's draw a table that describes the state of both mugs after each step:

	Milk	Tea	Milk	Tea
Initial state	x	0	0	x
Pour a spoon from right to left	x	s	0	$x - s$
Pour a spoon from left to right	$x - \frac{x}{x+s}s$	$s - \frac{s}{x+s}s$	$\frac{x}{x+s}s$	$x - s + \frac{s}{x+s}s$

If we simplify the expressions in the bottom row we get:

Milk	Tea	Milk	Tea
$\frac{x^2}{x+s}$	$\frac{s^2}{x+s}$	$\frac{s^2}{x+s}$	$\frac{x^2}{x+s}$

Hence the correct answer is C), i.e. the ratio of tea to milk in the mug on the left is the same as the ratio of milk to tea in the mug on the right.

Problem 25

Fedor has a cube. However, the cube is ugly so he enlarged one of its dimensions by 1 cm and he reduced one of the other dimensions by 1 cm. He kept the third dimension unchanged. By doing this, the volume of the cube decreased by 6 cm^3. What was the volume of the original cube in cm^3?

Solution 25

Let the length of the cube's edge be a. We were given that:

$$\begin{aligned}(a+1)\times(a-1)\times a+6 &= a\times a\times a\\ (a\times a-1)\times a+6 &= a\times a\times a\\ a\times a\times a-a+6 &= a\times a\times a\\ a &= 6.\end{aligned}$$

The length of the original cube's edge was 6 cm, therefore its volume was $6\times6\times6 = 216$ cm^3.

Problem 26

Jenny got a special calculator for her birthday. The calculator has only one button which always does the following with the number on the display: It multiplies the number on the display by itself, then subtracts the number n and overwrites the original number on the display by the newly formed number. Jenny pressed the button when her favourite number was on the display and she got back the number 44. If she pressed the button when there was her favourite number plus one on the display, she would get back the number 63. What is the value of the number n?

Solution 26

Let Jenny's favourite number be n. Then we were given that:

$$44 = j\times j - n\times n \quad (1)$$

$$63 = (j+1)\times(j+1) - n\times n = j\times j + 2\times j + 1 - n\times n \quad (2)$$

If we subtract the first equation from the second one we get:

$$63 - 44 = 2 \times j + 1$$
$$19 = 2 \times j + 1$$
$$j = 9$$

Once we know the value of Jenny's favourite number it is easy to calculate the value of $n = 37$.

Problem 27

When Tim was travelling to Poland, he watched the cars driving in the opposite direction. He was surprised that the cars always drove at the same constant speed as he did and he met them at regular intervals – he met one car every minute. After a while he found out that this was caused by the border patrol, where the cars were released with constant time spacing. What was the time difference in minutes between the releases of two consecutive cars that Tim met?

Solution 27

The cars that Tim meets are all spaced some distance d apart. When Tim meets a car, then him and the next car have to travel the distance d to meet. Since both have the same speed they will both have to travel a distance $d/2$.

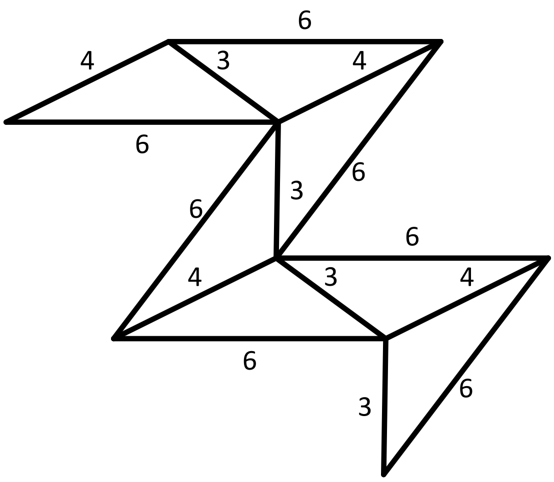
Let's imagine that Tim stopped his car exactly when he met a car in the opposite direction. Now the car going in the opposite direction has to travel twice the distance, because Tim is not going to meet it. That means the car will need 2 minutes to cover the whole distance d. This implies that the time between the release of two cars is two minutes.

Problem 28

Jonas has a building kit which contains seven wooden triangles, all of them with sides of lengths 3, 4 and 6 cm. Jonas started placing the triangles on the table in the following way: He placed every new triangle so that it touched one of the triangles already on the table with its entire side. He would like to place all the triangles touching each other on the table so that the formed 9-gon has the maximal circumference. What could be the largest circumference of the 9-gon in cm?

Solution 28

To get the largest possible circumference, we need to keep as many sides of length 6 cm at the circumference. We therefore place together triangles only by their sides of lengths 3 and 4 cm, which makes them form a chain of triangles as in the picture below. The total circumference is $7 \times 9 + 3 + 4 = 49$ cm.



Problem 29

Parents have seven children aged 1, 3, 5, 7, 9, 11 and 13. The sum of their ages is 49, which is a square (i.e. a number, that can be written as a product of two equal integers). In how many years will the sum of all the children's ages be again a square of some integer?

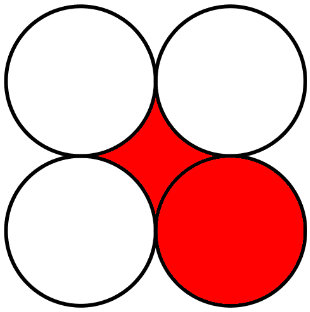
Solution 29

Since there are seven children, the sum of their ages increases by seven every year. That means in x years the sum of their ages will be $49+7x$ which can be expressed as $7\times(7+x)$. We would like this number to be the smallest possible square of a natural number. One key property of a squared number is that when you factorize it, each of the prime factors has to appear even number of times in the factorization. We therefore need each factor to appear even number of times in the product $7 \times (7 + x)$.

The expression $7+x$ must be therefore divisible by seven, so that seven appears twice in the factorization. The smallest possible is for $x = 7$, but then the factorization is $7 \times (7 + 7) = 7 \times 14 = 7 \times 7 \times 2$, which contains two only once. The second smallest possible is for $x = 14$, where we get factorization $7 \times 7 \times 3$. However, for the third smallest one for $x = 21$ we get the factorization$7 \times 7 \times 2 \times 2 = 196$, which is a square since $14 \times 14 = 196$. The answer is since in 21 years.

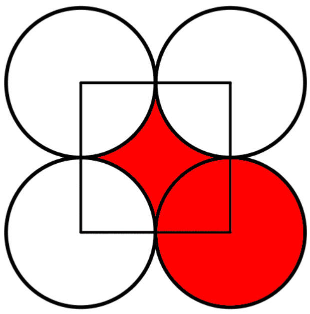
Problem 30

Dorothy has a cooker with four hot plates, each with 10 cm diameter. Dorothy wanted to cook carrot soup for lunch one day but she didn't watch it, the soup boiled over and spilled over the cooker area that is coloured red in the picture. What is the area of the cooker that the soup spilled on in cm^2?



Solution 30

If we draw a square that with its vertices in the centres of all the circles we see that the area of the red part is equal to (the area of the square) — (four times the area of a quarter of the circle). Hence the red part has area: $5 \times 5 \times \pi + 10 \times 10 - 4 \times 5 \times 5 \times \pi/4 = 25 \times \pi + 100 - 25 \times \pi = 100\ cm^2$.



Problem 31

Dorothy is messing around in the kitchen again. This time she took two 1-litre jugs and filled each with sorrel juice. The first jug has water to sorrel ratio of 3:1. The second jug has water to sorrel ratio of 4:1. Dorothy tasted the juice in both jugs but she didn't like the consistency of neither of them. So she took a pot and mixed the contents of both jugs in it. What is the ratio of water to sorrel in the pot?

Solution 31

The first jug has water to sorrel ratio of 3:1, the second jug has water to sorrel ratio of 4:1. Since both of them are one litre jugs, the first jug contains 750 ml of water and 250 ml of sorrel, the second jug contains 800 ml of water and 200 ml of sorrel. If we mix the contents of both jugs we get a liquid with $800 + 750 = 1550$ ml of water and $250 + 200 = 450$ ml of sorrel. After mixing, the water to sorrel ratio is therefore $1550{:}450 = 31{:}9$.

Problem 32

What is the sum of the number of all possible moves of knight for all possible squares on an 8×8 chessboard? Note: Knight moves to a square that is two squares away horizontally and one square vertically, or two squares vertically and one square horizontally. The complete move therefore looks like the letter "L".

Solution 32

To solve this problem we just need to draw an 8×8 chessboard and write down the number of possible knight's moves into each square. We then get the result by summing all 64 numbers we wrote down. The filled in chessboard looks like this:

2	3	4	4	4	4	3	2
3	4	6	6	6	6	4	3
4	6	8	8	8	8	6	4
4	6	8	8	8	8	6	4
4	6	8	8	8	8	6	4
4	6	8	8	8	8	6	4
3	4	6	6	6	6	4	3
2	3	4	4	4	4	3	2

The sum of all the numbers is $4 \times (1 \times 2 + 2 \times 3 + 5 \times 4 + 4 \times 6 + 4 \times 8) = 336$.

Problem 33

Indiana Jones is in an ancient maze which has a legendary treasure of immense value at its end. Indy knows that he is in the top left corner of the maze (labelled J) and the treasure (labelled P) is in the bottom right corner. Indy decides he will always move one square down or right (he won't go back up or left). He doesn't know though, that there is a trap in the maze (labelled X) which causes the treasure to cave in if stepped on. If Indy decides randomly in each step whether he goes down or right, what is the chance he gets the treasure? Submit the result as a percentage rounded to two decimal points. Note: Calculate the chance by dividing the number of paths containing the trap by the total number of paths through the maze.

J					
			X		
					P

Solution 33

Let's begin by counting the number of unique paths through the maze. We can do that by filling every square in the maze with the number of unique paths that go to this square and then reading the final number in the rightmost bottom square. But how do we fill all the squares? We start with a one in the leftmost top corner since Indy starts here and hence there is only one option how he could get there. Since Indy can move in the maze only right and down, he can reach the square A in the picture below only from square B or C. So the number of unique paths to the square A is the sum of the number of unique paths to the square B plus the number of unique paths to the square C.

...	...	...
...	...	C
...	B	A

We therefore fill in each square in the table by writing there the sum of the number in the square directly above it and the number in the square directly to the left. For squares at the edges we just copy the number from the left or from the top. The filled in table will look like this:

1	1	1	1	1	1
1	2	3	4	5	6
1	3	6	10	15	21
1	4	10	20	35	56
1	5	15	35	70	126

We still need to count the number of paths that contain the trap. There are ten paths to the square with the trap. But in how many ways can we complete those paths? We need to draw a similar table but we won't count the number of paths which don't contain the trap. It will look like this:

1	1	1	1	0	0
1	2	3	4	0	0
1	3	6	10	10	10
0	0	0	10	20	30
0	0	0	10	30	60

There are 60 unique paths that contain the trap. Indy's chance of getting the treasure is $(126-60)/126 = 66/126 = 52.4$ %.

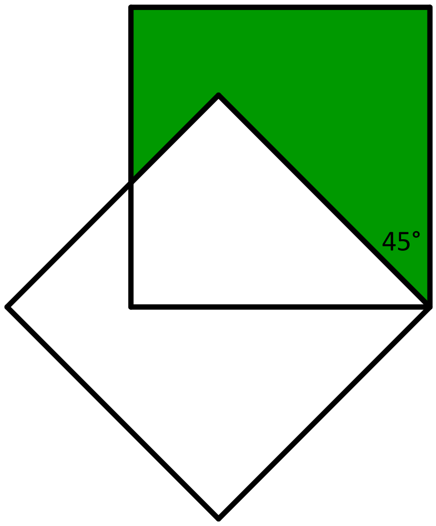
Problem 34

In the 2009 presidential election in Slovakia, Ivan Gašparovič won over Iveta Radičová. In the second round of the election Radičová got 988808 votes which was 44.47 % of the valid votes casted. Many people didn't go to vote though. How many people would have to come and vote for Radičová so that she would win the second round of the election? Round your answer up to thousands.

Solution 34

Radičová got 44.47 % of all valid votes casted, the total number of casted votes was therefore $988808/0.4447 = 2223539$ valid votes. Gašparovič therefore had to get $2223539 - 988808 = 1234731$ votes. Radičová would win is she got at least $1234731 - 988808 + 1 = 245924$ votes more than she got. If we round this number up to the closest thousand, we get 246000.

Problem 35
A square has a side of length 1 meter. We rotate an identical square around one of its vertices by 45 degrees as you can see in the picture. The original square is therefore divided into two parts – the green one and the white one. What is the area of the larger part in cm^2?



Solution 35
Let's add two extra segments that go through the point B and both are perpendicular to the sides of the square into the picture. We will label the intersections with the sides of the square as W, X, Y, Z, as you can see in the picture below. We were given that $|AB| = 1$ cm. Pythagorean theorem gives us that $|AX|^2 + |BX|^2 = |AB|^2 = 1$. Since the angles BAX and ABX are both 45°, the triangle BAX has to be isosceles, therefore $|AX| = |BX|$. That implies $2|AX|^2 = 1$, $|AX| = \sqrt{1/2} = \sqrt{2}/2$. Since $|BX| + |BY| = 1$, then $|BY| = 1 - \sqrt{2}/2$. Similarly to BAX, the triangle BYH is also isosceles, so $|HY| = |BY|$. Since the point B lies on the diagonal of the original square, it is the case that $|BY| = |BZ|$.
Now we know everything to calculate the area of the green part. It is equal to: $(\sqrt{2}/2 \times \sqrt{2}/2)/2 + (1 \times (1 - \sqrt{2}/2)) + ((1 - \sqrt{2}/2) \times (1 - \sqrt{2}/2))/2 = 2 - \sqrt{2}$. We still need to verify whether the area of the green part: $2 - \sqrt{2}$, or the area of the white part: $1 - (2 - \sqrt{2}) = \sqrt{2} - 1$ is larger. We easily find out that greater is $2 - \sqrt{2}$.